



Labor Input and Compost Output Using a Quadratic Production Function

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ABSTRACT

This study analyzes the relationship between workforce size and total compost output using a quadratic production function. A descriptive mathematical approach is applied to the function reported in a group project-based learning report on compost production. The coefficients are treated as fixed inputs, and the analysis consists of differentiation, a second-derivative test, and comparison of neighboring integer workforce levels. For the function $Q(x) = -0.5x^2 + 10x$, where x represents the number of workers, the maximum total output is 50 production units at 10 workers. Output at both 9 and 11 workers is 49.5 units. Marginal product decreases throughout the model domain, becomes zero at 10 workers, and turns negative beyond that point. Average output at the maximum is 5 units per worker, which is distinct from maximum average labor productivity. The analysis establishes an optimum conditional on the stated function; it does not demonstrate that 10 workers is an empirically validated or profit-maximizing staffing level. Applying the result requires confirmation of the output unit, production period, underlying observations, and model coefficients, alongside assessment of equipment capacity and compost quality.

Keywords: labor input; compost production; quadratic production function; output maximization

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INTRODUCTION

Compost production offers a way to recover organic materials while creating a soil amendment for agricultural use. Community and small-scale composting projects also require workable arrangements for collecting feedstock, organizing production, and distributing the finished product. Lim et al. (2019) assessed an integrated composting and organic farming project in Malaysia, while De Boni et al. (2022) evaluated environmental, economic, institutional, and social aspects of community composting. These studies show why production planning needs to account for the operating setting as well as the quantity produced.

Labor is needed for material handling, sorting, turning, screening, and packaging, but workforce size alone does not describe how efficiently those activities are performed. Yesaya et al. (2021) used activity-based costing to identify the time spent on composting

operations, including plastic removal. Oviedo et al. (2025) compared manual and mechanized mixing arrangements and found that technology choices changed labor demand. Their findings suggest that a staffing recommendation should be tied to a defined process and equipment configuration.

A quadratic production function provides a tractable way to examine an input–output relationship and calculate a turning point. Its usefulness depends on whether the imposed curve adequately represents the production system. Dhakal and Lange (2021) discuss the consequences of functional-form choice in agricultural input response models. In a composting context, Mason (2006) reviews models that represent biological and physical processes. Such process models differ from a single-input workforce function: they help explain decomposition, whereas a labor–output equation summarizes a proposed production relationship.

This study examines a quadratic function reported for a small-scale compost production unit. The objective is to identify the workforce size that maximizes total output and clarify the interpretation of that solution. The analysis distinguishes total output, marginal product, and average labor productivity, then considers the information needed before the result can support an operational staffing decision.

METHODS

The study uses a descriptive quantitative approach to analyze a quadratic production function from a group project-based learning report on compost fertilizer optimization. The unit of interest is a single small-scale compost production operation. The analysis evaluates the reported function algebraically; it does not estimate its coefficients from a newly analyzed dataset or test a causal effect of labor input.

The production function is:

$$Q(x) = -0.5x^2 + 10x \quad (1)$$

Here, x denotes the number of workers and $Q(x)$ denotes total compost output in production units. The physical output unit and the length of the production period must be specified before the predicted quantities can be interpreted as operational capacity. For the present calculations, the output unit and period are held constant across workforce levels. Worker-hours, skill differences, equipment, and feedstock quantities are not separate variables in the function.

The quadratic is evaluated over $0 \leq x \leq 20$, the interval in which both labor input and predicted output are nonnegative. This interval follows from $Q(x) = 0.5x(20 - x)$; it is a mathematical restriction rather than an observed operating range. Workforce size is temporarily treated as continuous for differentiation and then restricted to integer values when comparing staffing levels.

The first derivative gives the marginal product of labor, and the second derivative determines the curvature:

$$Q'(x) = -x + 10 \quad (2)$$

$$Q''(x) = -1 \quad (3)$$

The stationary point is found by setting $Q'(x) = 0$. The second-derivative test is used to determine whether it is a maximum. Direct substitution at neighboring integer values provides an additional check because a workforce cannot contain a fractional worker in this formulation. Table 1 reports calculated outputs and productivity measures; no row is presented as a field observation.

Average output per worker is calculated separately for positive workforce size:

$$AP(x) = Q(x)/x = 10 - 0.5x, x > 0 \quad (4)$$

This measure is used to distinguish an output maximum from an average-productivity maximum. No cost, revenue, or profit values are estimated because the function contains neither wage rates nor compost prices. The discussion draws on published composting studies to identify operational conditions that would need to be measured in a subsequent validation.

RESULT AND DISCUSSION

Solving $-x + 10 = 0$ gives a stationary point at $x = 10$. Since $Q''(x) = -1$ is negative throughout the domain, this point is the unique maximum of the quadratic. Substitution gives $Q(10) = -0.5(10)^2 + 10(10) = 50$ production units. The endpoint values $Q(0)$ and $Q(20)$ are both zero. These results establish the mathematical maximum of the function without implying that its coefficients have been validated against production records.

Table 1 compares selected workforce levels. At 9 and 11 workers, the function gives the same total output of 49.5 units, below the value of 50 at 10 workers. The integer comparison therefore agrees with the calculus result. Increasing the workforce from 9 to 10 adds 0.5 units, while increasing it from 10 to 11 subtracts 0.5 units. These finite changes should be distinguished from the derivative, which describes the local rate of change in the continuous model.

Table 1. Calculated output and average productivity near the model maximum

Workers, x	Total output, $Q(x)$	Marginal product, $Q'(x)$	Average output per worker
5	37.5	5	7.5
8	48	2	6
9	49.5	1	5.5
10	50	0	5
11	49.5	-1	4.5
12	48	-2	4
15	37.5	-5	2.5

Note. All entries are derived from the supplied function; they are not field observations.

The marginal product falls as x increases, including at workforce levels below 10. Diminishing marginal product therefore occurs throughout this quadratic, not only after the turning point. Total output continues to rise while marginal product is positive and declines only when marginal product becomes negative. The model does not contain an initial region of increasing marginal returns.

Average output per worker also declines throughout the positive domain because $AP'(x) = -0.5$. At 9 workers, average output is 5.5 units per worker; at the output maximum of 10 workers, it is 5 units per worker. Thus, the total-output maximum is not the maximum of average labor productivity. Nor is it a measure of technical efficiency relative to a benchmark production frontier. Keeping these concepts separate prevents the turning point from being interpreted as evidence of every form of production efficiency.

The result is sensitive to the specification of the production function. Cerrato and Blackmer (1990) found that alternative crop-response functions could imply different economic input recommendations even when common fit statistics were similar. Their study concerns fertilizer application to corn, so its numerical findings cannot be transferred to compost labor allocation. Its methodological implication is relevant: selecting a convenient quadratic does not establish its empirical adequacy. The workforce function should be compared with plausible alternatives once paired labor and output observations are available.

Composting models also illustrate the importance of validating the process represented. Gomes and Pereira (2008) compared a model of composting reactions with experimental observations. The present workforce equation instead omits decomposition time and material characteristics. It can describe a conditional output pattern, but cannot show whether adding a worker accelerates biological stabilization or removes a particular processing bottleneck. Claims that crowding, tool shortages, or coordination problems cause the decline beyond 10 workers remain hypotheses for field investigation.

Turning practices provide an example of a task that links labor use to process conditions. Kalamdhad and Kazmi (2009) examined compost stability and chemical characteristics under different turning frequencies in a rotary drum. Getahun et al. (2012) investigated turning frequency in municipal solid waste composting. These studies support recording the work performed, not merely headcount. Their results do not identify a universal worker requirement, because the equipment and materials differ from the production unit considered here.

Additional work can also change product quality and environmental performance. Ma et al. (2022) found that turning frequency affected ammonia emissions and nitrogen loss during chicken manure composting. Tang et al. (2023) examined moisture, aeration, and carbon-to-nitrogen ratio in kitchen waste composting, while Peng et al. (2023) considered aeration patterns and turning frequency alongside maturity and gaseous emissions. Consequently, output comparisons should use compost meeting a consistent quality criterion. Counting material before comparable stabilization could make a faster operation appear more productive without demonstrating an equivalent finished product.

Equipment and process control may change the relationship between labor and output. Chen (2016) compared food-waste composting costs across facilities and discussed the role of automation. Oliveri et al. (2023) assessed monitoring and control improvements through process simulation and economic analysis. Together with the labor-demand comparison reported by Oviedo et al. (2025), these studies suggest that coefficients estimated for one operating arrangement may not remain applicable after mechanization or process redesign. This is a reason to recalibrate a workforce model when production conditions change.

Economic decisions require a broader accounting boundary. Pandyaswargo and Premakumara (2014) assessed how composting scale relates to financial sustainability. Wang et al. (2023) compared decentralized technologies and identified the importance of operating costs and waste-handling subsidies. Banunle et al. (2024) evaluated small-scale composting through cost–benefit analysis and sensitivity tests. These approaches include information absent from the current output equation. The 10-worker solution therefore cannot establish whether the additional output pays for the labor needed to produce it.

A full assessment should also consider costs and impacts outside the immediate production stage. Liao et al. (2023) evaluated construction, operation, and demolition of a decentralized composting facility using life-cycle assessment and costing. De Rosa et al. (2022) examined environmental and economic trade-offs in supplying and using composted manure. Their findings show that an environmental improvement need not coincide with the lowest financial cost. Maximizing the quantity predicted by a workforce equation cannot, by itself, establish the sustainability of the composting system.

The principal limitation is the lack of a documented calibration and validation procedure for the quadratic coefficients. Field application requires paired observations of labor and comparable finished output, a stated measurement period, and records of worker-hours, equipment capacity, feedstock, and process conditions. Repeated production cycles would help assess variability. Until those observations are analyzed, 10 workers should be treated as the optimum of the stated equation rather than a general staffing recommendation for small-scale composting.

CONCLUSION

The quadratic function $Q(x) = -0.5x^2 + 10x$ has a unique total-output maximum of 50 production units at 10 workers. The negative second derivative and comparison with neighboring integer values confirm this solution. Marginal product declines throughout the model domain, while total output declines only beyond 10 workers. Average output at the maximum is 5 units per worker and is not itself maximized. The analysis provides a mathematical basis for examining labor allocation, but operational recommendations require field validation, defined output units and production periods, and explicit consideration of costs and compost quality.

BIBLIOGRAPHY

- Banunle, A., Fei-Baffoe, B., Miezah, K., Ewusi-Mensah, N., Jørgensen, U., Aidoo, R., Amoah, A., Addo-Fordjour, P., & Abaidoo, R. C. (2024). Economic viability assessment of small-scale biomass composting project within a developing country context. *Circular Economy and Sustainability*, 4(2), 951–971. <https://doi.org/10.1007/s43615-023-00328-4>
- Cerrato, M. E., & Blackmer, A. M. (1990). Comparison of models for describing corn yield response to nitrogen fertilizer. *Agronomy Journal*, 82(1), 138–143. <https://doi.org/10.2134/agronj1990.00021962008200010030x>
- Chen, Y.-T. (2016). A cost analysis of food waste composting in Taiwan. *Sustainability*, 8(11), Article 1210. <https://doi.org/10.3390/su8111210>

- De Boni, A., Melucci, F. M., Acciani, C., & Roma, R. (2022). Community composting: A multidisciplinary evaluation of an inclusive, participative, and eco-friendly approach to biowaste management. *Cleaner Environmental Systems*, 6, Article 100092. <https://doi.org/10.1016/j.cesys.2022.100092>
- De Rosa, D., Biala, J., Nguyen, T. H., Mitchell, E., Friedl, J., Scheer, C., Grace, P. R., & Rowlings, D. W. (2022). Environmental and economic trade-offs of using composted or stockpiled manure as partial substitute for synthetic fertilizer. *Journal of Environmental Quality*, 51(4), 589–601. <https://doi.org/10.1002/jeq2.20255>
- Dhakal, C., & Lange, K. (2021). Crop yield response functions in nutrient application: A review. *Agronomy Journal*, 113(6), 5222–5234. <https://doi.org/10.1002/agj2.20863>
- Getahun, T., Nigusie, A., Entele, T., Van Gerven, T., & Van der Bruggen, B. (2012). Effect of turning frequencies on composting biodegradable municipal solid waste quality. *Resources, Conservation and Recycling*, 65, 79–84. <https://doi.org/10.1016/j.resconrec.2012.05.007>
- Gomes, A. P., & Pereira, F. A. (2008). Mathematical modelling of a composting process, and validation with experimental data. *Waste Management & Research*, 26(3), 276–287. <https://doi.org/10.1177/0734242x07086514>
- Kalamdhad, A. S., & Kazmi, A. A. (2009). Effects of turning frequency on compost stability and some chemical characteristics in a rotary drum composter. *Chemosphere*, 74(10), 1327–1334. <https://doi.org/10.1016/j.chemosphere.2008.11.058>
- Liao, N., Lü, F., Zhang, H., Shao, L., & He, P. (2023). Environmental and economic assessment of the construction, operation, and demolition of a decentralized composting facility. *Science of The Total Environment*, 884, Article 163724. <https://doi.org/10.1016/j.scitotenv.2023.163724>
- Lim, L. Y., Lee, C. T., Bong, C. P. C., Lim, J. S., & Klemeš, J. J. (2019). Environmental and economic feasibility of an integrated community composting plant and organic farm in Malaysia. *Journal of Environmental Management*, 244, 431–439. <https://doi.org/10.1016/j.jenvman.2019.05.050>
- Ma, Q., Li, Y., Xue, J., Cheng, D., & Li, Z. (2022). Effects of turning frequency on ammonia emission during the composting of chicken manure and soybean straw. *Molecules*, 27(2), Article 472. <https://doi.org/10.3390/molecules27020472>
- Mason, I. G. (2006). Mathematical modelling of the composting process: A review. *Waste Management*, 26(1), 3–21. <https://doi.org/10.1016/j.wasman.2005.01.021>
- Oliveri, L. M., Arfò, S., Matarazzo, A., D'Urso, D., & Chiacchio, F. (2023). Improving the composting process of a treatment facility via an Industry 4.0 monitoring and control solution: Performance and economic feasibility assessment. *Journal of Environmental Management*, 345, Article 118776. <https://doi.org/10.1016/j.jenvman.2023.118776>

- Oviedo, A., Mora, M., Ponsa, S., & Colón, J. (2025). Techno-economic evaluation of decentralized community composting as a management model to valorize organic matter in small and medium-sized towns. *Journal of Material Cycles and Waste Management*, 27(6), 4234–4251. <https://doi.org/10.1007/s10163-025-02340-2>
- Pandya Swargo, A. H., & Premakumara, D. G. J. (2014). Financial sustainability of modern composting: the economically optimal scale for municipal waste composting plant in developing Asia. *International Journal of Recycling of Organic Waste in Agriculture*, 3(3), Article 4. <https://doi.org/10.1007/s40093-014-0066-y>
- Peng, L., Tang, R., Wang, G., Ma, R., Li, Y., Li, G., & Yuan, J. (2023). Effect of aeration rate, aeration pattern, and turning frequency on maturity and gaseous emissions during kitchen waste composting. *Environmental Technology & Innovation*, 29, Article 102997. <https://doi.org/10.1016/j.eti.2022.102997>
- Tang, R., Liu, Y., Ma, R., Zhang, L., Li, Y., Li, G., Wang, D., Lin, J., Li, Q., & Yuan, J. (2023). Effect of moisture content, aeration rate, and C/N on maturity and gaseous emissions during kitchen waste rapid composting. *Journal of Environmental Management*, 326, Article 116662. <https://doi.org/10.1016/j.jenvman.2022.116662>
- Wang, H., Qin, Y., Xin, L., Zhao, C., Ma, Z., Hu, J., & Wu, W. (2023). Preliminary techno-economic analysis of three typical decentralized composting technologies treating rural kitchen waste: a case study in China. *Frontiers of Environmental Science & Engineering*, 17(4), Article 47. <https://doi.org/10.1007/s11783-023-1647-7>
- Yesaya, M., Mpanang'ombe, W., & Tilley, E. (2021). The cost of plastics in compost. *Frontiers in Sustainability*, 2, Article 753413. <https://doi.org/10.3389/frsus.2021.753413>