



Workforce Size and Compost Production Output Using a Quadratic Production Model

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ABSTRACT

This study examines the workforce size that maximizes total output in a quadratic model of compost production. The source manuscript attributes the production function to a group report on compost production optimization, but does not provide the underlying observations or coefficient estimation procedure. The analysis therefore evaluates the supplied model rather than estimates an empirical effect of task allocation. The production function is $Q(x) = -0.5x^2 + 10x$, where x denotes the number of workers and $Q(x)$ denotes output in unspecified production units. Differentiation gives a stationary point at 10 workers, and the negative second derivative confirms a maximum of 50 units. Direct comparison with 9 and 11 workers, each yielding 49.5 units, confirms the integer solution. Average output at the maximum is 5 units per worker; this does not represent the maximum of average labor productivity. The result is conditional on the supplied coefficients and does not establish a statistically significant effect of division of labor. Field validation requires defined output units, a common production period, workforce records, and information on task allocation and production constraints.

Keywords: workforce size; compost production; quadratic production function; labor productivity

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INTRODUCTION

Compost production can convert locally available organic materials into a useful agricultural input. Community programs have introduced composting through practical training and the use of local organic residues (Hamzah et al., 2020; Suhastyo, 2017). Production involves tasks such as collecting materials, preparing the mixture, turning, and packaging. Planning the workforce for these tasks requires a clear distinction between the number of workers and the way tasks are assigned to them.

Composting also involves biological processes that cannot be represented by workforce size alone. Mathematical models reviewed by Mason (2006) address process behavior, while Gomes and Pereira (2008) developed a composting model validated against

experimental observations. These studies provide a basis for distinguishing a simplified production relationship from a model of the physical and biological composting process. A workforce-output function can illustrate an allocation decision, but its numerical predictions require validation in the production setting where it is applied.

This study examines the quadratic production function supplied in a group-report-based manuscript. Its objective is to determine the workforce size that maximizes total output and to distinguish that solution from average labor productivity. The analysis also considers what the model can establish about labor allocation. Because task specialization is not represented by a separate variable, the model cannot isolate the effect of division of labor.

METHODS

The study uses descriptive mathematical analysis of a production function attributed in the source manuscript to a group report on compost production optimization. The report's title, authors, observation period, and underlying dataset were not supplied. The coefficients are consequently treated as given model inputs; they are not presented as estimates obtained in this study. No regression, hypothesis test, or field experiment is reported.

The supplied production function is:

$$Q(x) = -0.5x^2 + 10x \quad (1)$$

Here, x is workforce size and $Q(x)$ is total output. The physical output unit and the production period are unspecified, so output is retained as 'production units' without conversion to kilograms, bags, or daily output. For calculus, x is treated as a continuous variable. The resulting candidate solution is then checked against neighboring integer workforce sizes.

For nonnegative output and nonnegative labor input, the mathematical domain is $0 \leq x \leq 20$ because $Q(x) = 0.5x(20 - x)$. This is a domain restriction implied by the polynomial, not an empirically validated operating range. Predictions beyond 20 workers would be negative and are not physically meaningful as production quantities.

The marginal product of labor and the second derivative are:

$$Q'(x) = -x + 10 \quad (2)$$

$$Q''(x) = -1 \quad (3)$$

Setting $Q'(x) = 0$ gives $x = 10$. Since $Q''(x)$ is negative throughout the domain, the stationary point is the unique maximum of the supplied quadratic. Substitution gives $Q(10) = -0.5(10)^2 + 10(10) = 50$. A direct integer comparison verifies that 10 workers also maximizes the function when workforce size must be a whole number.

Average labor productivity is defined separately for $x > 0$:

$$AP(x) = Q(x)/x = 10 - 0.5x, \quad x > 0 \quad (4)$$

This distinction is necessary because maximizing total output and maximizing output per worker are different objectives. The analysis uses the same output unit and period for every model evaluation; actual working hours and worker skill differences are not represented.

RESULT AND DISCUSSION

The supplied function reaches its maximum total output at 10 workers. Table 1 compares selected workforce levels around this point. The entries are calculated directly from the stated function and are not observed production records. At 9 and 11 workers, total output is 49.5 units, compared with 50 units at 10 workers. Thus, the continuous maximum is also the integer maximum.

Table 1. Calculated output and average productivity near the model maximum

Workers, x	Total output, $Q(x)$	Marginal product, $Q'(x)$	Average output per worker
8	48	2	6
9	49.5	1	5.5
10	50	0	5
11	49.5	-1	4.5
12	48	-2	4

Note. All entries are derived from the supplied function; they are not field observations.

Moving from 9 to 10 workers increases modeled output by 0.5 units, whereas moving from 10 to 11 workers reduces it by 0.5 units. The marginal product $Q'(x)$ is positive below 10, zero at 10, and negative above 10. Its derivative is -1 throughout the domain. Accordingly, diminishing marginal product occurs throughout the model, while declining total output begins only after the maximum.

Average productivity at 10 workers is $Q(10)/10 = 5$ units per worker. At 9 workers it is 5.5 units per worker, despite the lower total output. More generally, $AP(x) = 10 - 0.5x$ decreases as workforce size increases. The 10-worker solution therefore maximizes total output, not output per worker. Within the purely mathematical integer domain, average productivity is highest at one worker, but this is not a staffing recommendation because the model contains no minimum staffing or task feasibility requirements.

The maximum can also be written in vertex form as $Q(x) = 50 - 0.5(x - 10)^2$. This expression shows that any departure from 10 workers reduces the predicted total by one-half of the squared workforce deviation. It describes the shape imposed by the model and should not be interpreted as an empirically measured penalty for understaffing or overstaffing.

The source narrative associates lower output beyond 10 workers with workspace limitations, equipment shortages, and coordination difficulties. These are possible explanations for investigation, but none appears as a measured variable in the production function. Likewise, collecting, turning, and packaging are not separately modeled. The calculation therefore cannot determine the best assignment of workers across production stages or establish that specialization caused an increase in output.

Evidence on compost production illustrates why these omitted conditions matter. Nina Veronika and Umar Linggom (2024) examined compost made from oil palm trunk waste with additional materials and assessed chemical and moisture-related parameters. Ramli and Wahab (2022) investigated compost preparation using biohydrogen residue and goat manure, with observations of temperature, pH, moisture, and nutrient content. These studies concern particular materials and processes; their findings do not validate the

workforce coefficients used here. They indicate variables that a field evaluation should document alongside labor input.

Validation should link a stated workforce size to a defined output quantity over a comparable production period. Gomes and Pereira (2008) demonstrate the importance of comparing a composting model with experimental data, whereas the present function is supplied without that calibration record. A useful extension would record worker-hours and task assignments together with input materials, equipment availability, processing time, and product quality. These observations would permit a test of whether the quadratic relationship describes the production system adequately.

The output-maximizing solution is also distinct from a profit-maximizing solution. Wages, compost prices, and other production costs are absent from the model, so profitability cannot be calculated. Nor does total output establish compost maturity or suitability for agricultural use. The result should be used as a conditional mathematical illustration of workforce planning, with operational and economic decisions reserved for analysis supported by local data.

CONCLUSION

For the supplied function $Q(x) = -0.5x^2 + 10x$, total output is maximized at 10 workers and equals 50 production units. The negative second derivative and comparison with neighboring integer values confirm this result. Average labor productivity at that point is 5 units per worker and is not itself maximized. The analysis does not establish a causal or statistically significant effect of division of labor. Before the numerical optimum is used for staffing decisions, the source data, output unit, production period, and model coefficients require confirmation and field validation.

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