



Marginal Cost Integration and Fixed Cost Sensitivity in an Illustrative Farming Model

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ABSTRACT

This study examines the reconstruction of total farming costs from an assumed marginal cost function and distinguishes minimum average cost from profit maximization. A deterministic analytical model uses $MC(Q) = 5Q + 100$ and integrates this expression to obtain $TC(Q) = 2.5Q^2 + 100Q + F$, where F is fixed cost. Two fixed-cost scenarios from the source manuscript, IDR 1,000,000 and IDR 7,750,000, are evaluated without treating them as validated farm observations. The corresponding minimum-average-cost outputs are approximately 632.46 and 1,760.68 units. The previously stated output of 20 units would require $F = \text{IDR } 1,000$ under the same monetary scale. A profit-maximizing output of 991.2 units implies a constant selling price of IDR 5,056 per unit. At that price, modeled profit is IDR 1,456,193.60 in the lower-cost scenario and a loss of IDR 5,293,806.40 in the higher-cost scenario. Fixed cost changes average-cost minima and profit levels but not the interior output choice when marginal cost and price are held constant. The results clarify the model's internal relationships; they do not establish an optimal production recommendation for a particular farm. Empirical application requires defined output units, consistent accounting periods, documented cost and price data, and production constraints.

Keywords: *marginal cost; total cost; average cost; fixed cost; farm planning*

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INTRODUCTION

Farm cost analysis involves several distinct questions: how total expenditure changes with output, which output minimizes average cost, and which output maximizes profit at a given price. These questions require different calculations. Integrating a marginal cost function reconstructs variable cost, but total cost also requires a boundary condition representing fixed cost. Ramadhanty et al. (2023) discuss integral methods in economic analysis, providing a conceptual starting point for this worked example.

Empirical farm studies demonstrate the importance of identifying the production setting and accounting basis. Laiya et al. (2017) distinguish fixed costs, variable costs, and income in hybrid maize farming. Nirmala and Hardjanto (2022) combine production-function estimation, marginal-product analysis, and revenue-to-cost ratios in a coffee-farm study.

These approaches address observed farming conditions and cannot be replaced by an uncalibrated cost equation.

This study evaluates a hypothetical linear marginal cost function supplied in the source manuscript. The objectives are to derive total and average costs, verify the reported efficiency and profit-maximizing outputs, and examine sensitivity to fixed cost. The contribution is a reproducible analytical example and correction of inconsistent numerical interpretations. It is not a field validation, a new integration method, or an evaluation of farmer empowerment.

METHODS

The study is a deterministic mathematical analysis supported by selected literature. It does not use a documented farm-level dataset or a purposively sampled research location. The marginal cost coefficients are assumptions: no estimation sample, commodity-specific calibration, or uncertainty measures were supplied. Q denotes a nonnegative, continuously divisible output quantity; the physical unit and accounting period remain unspecified.

For the numerical scenarios, all monetary quantities are expressed in rupiah on the same scale. Marginal cost is measured in IDR per output unit. The coefficient 5 therefore has units of IDR per squared output unit, and 100 has units of IDR per output unit. Equation (1) defines the assumed marginal cost function.

$$MC(Q) = 5Q + 100, \quad Q \geq 0 \quad (1)$$

Let F denote fixed cost over the modeled accounting period. Using $TC(0) = F$ gives Equation (2). This boundary condition is essential because differentiation removes the constant term. The marginal cost function alone cannot identify F .

$$TC(Q) = \int (5Q + 100) dQ = 2.5Q^2 + 100Q + F \quad (2)$$

The source manuscript states fixed-cost values of IDR 1,000,000 and IDR 7,750,000. Both are retained as scenario inputs. The higher value is not treated as an independently verified observation because its specific source, farm scale, and accounting period were not identified. An additional $F = \text{IDR } 1,000$ calculation diagnoses the origin of the stated 20-unit average-cost minimum; it is not a third empirical farm.

For $Q > 0$, average cost is total cost divided by output. Differentiating it gives the minimum-average-cost condition in Equation (3). Because F is positive, the second derivative of average cost is positive on this domain. Average cost also tends to infinity as Q approaches zero or grows without bound, so the stationary point is its unique global minimum.

$$AC(Q) = 2.5Q + 100 + F/Q, \quad AC'(Q) = 2.5 - F/Q^2 = 0 \quad (3)$$

Profit maximization assumes a constant output price P , sale of all output, and no capacity or financing constraint. Revenue is PQ . The profit function and optimal nonnegative output are given in Equation (4). The second derivative of profit is -5 , establishing strict concavity. For $P \leq 100$, the maximum under these assumptions is at $Q = 0$.

$$\pi(Q) = PQ - TC(Q), \quad Q^* = \max\{0, (P - 100)/5\} \quad (4)$$

The source reports $Q = 991.2$ but does not document its associated market-price observation. Substitution into $P = MC(Q)$ implies $P = \text{IDR } 5,056$ per unit. This reverse-

calculated value is used only to reproduce the reported output. No claim is made that it represents an observed farm-gate price.

Sensitivity calculations vary F while holding the price and marginal cost function constant. Unrounded values are used in the calculations; tables display rounded outputs and monetary values. No statistical sensitivity analysis, confidence interval, or predictive validation is possible from the supplied assumptions alone.

RESULT AND DISCUSSION

Integration gives $TC(Q) = 2.5Q^2 + 100Q + F$. The term $2.5Q^2 + 100Q$ represents variable cost and F shifts the entire total-cost curve vertically. Differentiating this expression recovers the assumed marginal cost, confirming the internal consistency of the derivation.

Table 1 reports the minimum-average-cost outputs. For $F = \text{IDR } 1,000,000$, the minimum occurs at approximately 632.46 units, with average cost of IDR 3,262.28 per unit. For $F = \text{IDR } 7,750,000$, the minimum shifts to approximately 1,760.68 units, with average cost of IDR 8,903.41 per unit.

Table 1. Fixed cost and the minimum of average cost

Fixed cost F (IDR)	Minimum-cost output	Minimum AC (IDR/unit)
1,000 (diagnostic)	20.00	200.00
1,000,000	632.46	3,262.28
7,750,000	1,760.68	8,903.41

Note. Scenario calculations; output is continuously divisible and figures are rounded.

The 20-unit result is consistent with $F = \text{IDR } 1,000$, not $\text{IDR } 1,000,000$, when the coefficients retain their stated numerical values in rupiah. A different monetary scale could change the interpretation, but it would require rescaling all monetary coefficients consistently. No such rescaling is documented. The corrected results therefore use a single rupiah basis.

At the average-cost minimum, MC equals AC . The location depends on F because average fixed cost is F/Q . A larger fixed cost requires a greater output to offset its contribution per unit, given the same rising marginal cost. This relationship describes the assumed cost curve; it does not establish technical efficiency relative to other farms or long-run economies of scale.

Table 2 evaluates the reported output of 991.2 units at its implied price of IDR 5,056 per unit. Revenue and variable cost are the same in both fixed-cost scenarios. Profit differs by exactly IDR 6,750,000, the difference between the two fixed costs.

Table 2. Profit at $Q = 991.2$ and $P = \text{IDR } 5,056$ per unit

Quantity (IDR)	$F = 1,000,000$	$F = 7,750,000$
Revenue	5,011,507.20	5,011,507.20
Variable cost	2,555,313.60	2,555,313.60
Total cost	3,555,313.60	10,305,313.60
Profit	1,456,193.60	-5,293,806.40

Note. Scenario calculations; output is continuously divisible and figures are rounded.

In the lower fixed-cost scenario, the maximum modeled profit is IDR 1,456,193.60. In the higher fixed-cost scenario, the maximum is negative: –IDR 5,293,806.40. Thus, a first-order profit-maximization condition does not ensure a profitable business. A mathematical maximum may be the smallest attainable loss within the stated feasible set.

If fixed cost is unavoidable over the modeled period, producing 991.2 units in the higher-cost scenario loses less than producing nothing, which would yield a loss of IDR 7,750,000. The contribution to covering fixed cost is IDR 2,456,193.60. This is a short-run comparison conditional on unavoidable F ; it is not advice to continue a real loss-making farm. If costs are avoidable on exit, or production can be postponed, the decision problem must represent those alternatives explicitly.

The output minimizing average cost and the output maximizing profit need not coincide. Their equality requires P to equal the minimum average cost. At a higher price, the interior profit-maximizing output lies above the average-cost minimum; at a lower price, it lies below it. The different outputs in this example reflect different objectives and assumptions rather than a contradiction in economic theory.

More generally, $Q^* = (P - 100)/5$ for $P > 100$, while the average-cost minimum is $\sqrt{F/2.5}$. Hence the interior profit-maximizing output has zero sensitivity to F under the maintained assumptions, whereas the average-cost minimum rises with F . Maximum profit decreases one-for-one with an increase in fixed cost. These relationships would change if fixed-cost choices altered technology, productive capacity, or marginal cost.

Literature on mathematical cost analysis can clarify the relationship between marginal and total quantities (Ramadhanty et al., 2023). Nevertheless, the algebra cannot supply missing observations. The maize study by Laiya et al. (2017), for example, reports a specific local cost structure rather than the IDR 7,750,000 scenario used here. Its data should not be transferred to this equation without matching the farm, area, output definition, and period.

Similarly, Nirmala and Hardjanto (2022) examine production inputs and income using observed coffee-farm information. Their input-efficiency analysis differs from finding the minimum of an assumed average-cost function. A farmer may face land, labor, seasonal, or market constraints even when a smooth unconstrained equation suggests a larger output. The present model includes none of those constraints.

For practical development, records should identify the commodity, output unit, cultivated area, accounting period, fixed-cost components, and variable inputs. Cost coefficients should then be estimated or justified from those records, and model performance assessed on independent observations. Prices and feasible capacity should be documented before calculating an output recommendation. Any educational application with farmers would also require its own intervention design and outcome assessment; no such activity is reported here.

CONCLUSION

The assumed marginal cost function integrates to $TC(Q) = 2.5Q^2 + 100Q + F$. With consistent rupiah units, fixed costs of IDR 1,000,000 and IDR 7,750,000 produce minimum-average-cost outputs of approximately 632.46 and 1,760.68 units, respectively. The 20-unit result requires a different fixed cost of IDR 1,000.

The reported profit-maximizing output of 991.2 units is reproducible only by specifying its implied price of IDR 5,056 per unit. At that price, the lower fixed-cost scenario earns a positive modeled profit, whereas the higher-cost scenario incurs a loss. Fixed cost affects profitability and the average-cost minimum, but does not affect the interior profit-maximizing output when price and marginal cost are held constant.

These findings support careful separation of mathematical consistency from empirical validity. The model may serve as an instructional example of cost integration and optimization. Its use for actual farming decisions requires calibrated data, explicit constraints, and consistent definitions of output and cost.

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